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On decoupled theories in (5+1) dimensions from (F, D1, NS5, D5) supergravity configuration

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Abstract: It is well-known that $(N,\, M)$ 5-branes of type IIB supergravity form a non-threshold bound state with $(N',\, M')$ strings called the (F, D1, NS5, D5) bound state where the strings lie along one of the spatial directions of the 5-branes (hep-th/9905056). By taking low energy limits in appropriate ways on this supergravity configuration, we obtain the supergravity duals of various decoupled theories in (5+1) dimensions corresponding to noncommutative open string (NCOS) theory, open D-string (OD1) theory and open D5-brane (OD5) theory. We then study the $\mathrm{SL}(2,\mathbb{Z})$ transformation properties of these theories. We show that when the asymptotic value of the axion (χ_0) is rational (for which χ_0 can be put to zero), NCOS theory is always related to OD1 theory by strong-weak or S-duality symmetry. We also discuss the self-duality conjecture (hep-th/0006062) of both NCOS and OD1 theories. On the other hand, when χ_0 is irrational, we find that $\mathrm{SL}(2,\mathbb{Z})$ duality on NCOS theory gives another NCOS theory with different values of the parameters, but for OD1 theory $\mathrm{SL}(2,\mathbb{Z})$ duality always gives an NCOS theory. $\mathrm{SL}(2,\mathbb{Z})$ transformation on OD5 theory reveals that it gives rise to Little String Theory (LST) when $\chi_0 =$ rational, but it gives another OD5 theory with different values of the parameters when χ_0 is irrational.

Keywords: p-branes, D-branes, String Duality, AdS-CFT Correspondence.

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Contents

1. Introduction

Dynamical theories without gravity appear as particular low energy limits, known as decoupling limits, in string/M theory [1, 2]. These are either local field theories or non-local theories with or without noncommutative space-space and/or space-time structures in diverse dimensions having 16 or less supercharges and depend upon the kind of string/M theory vacua chosen and how the low energy limits are taken. Examples are the classic AdS/CFT correspondence of Maldacena [3] and their variations [1]. Low energy string/M theory in some sense plays complementary roles to these dynamical theories on the branes and vice-versa and has been proved to be quite fruitful in the recent past. These generalized correspondences are believed to shed light on QCD-like theories and in turn will help us to have the eventual formulation of M theory.

One such correspondence is obtained by considering a particular type IIB string theory vacuum consisting of D5-branes in the presence of an electric field along the brane [4]. The corresponding supergravity solution is described by (F, D5) bound state [5] where the fundamental strings lie along one of the spatial directions of D5-branes. The low energy limit or the decoupling limit is taken in such a way that the electric field attains a critical value and almost balances the tension of the F-strings. The resulting worldvolume theory on the brane decouples from the bulk closed string modes and becomes a non-gravitational non-local theory known as the NCOS theory in (5+1) dimensions. The (F, D5) supergravity solution in this limit [6] becomes the supergravity dual description of NCOS theory.[1]

By S-duality of type IIB supergravity (F, D5) solution goes over to (D1, NS5) solution [9, 10], so it is inferred that NS5-branes in the presence of a near critical RR 2-form field strength would similarly give rise to another decoupled non-local theory called the OD1 theory in (5+1) dimensions [4]. Here also, in the low energy limit, the 2-form field strength attains a critical value which almost balances the tension of D-strings and the resulting theory on the NS5-brane decouples from the bulk closed string modes and contains fluctuating light open D-strings in its spectra. The (D1, NS5) supergravity solution in the decoupling limit [9, 11, 12, 13] describes the supergravity dual of OD1 theory.

It is known that these two theories are related by strong-weak duality symmetry [4], where the coupling constant and the length scales in these two theories are related by $G_o^2 = 1/G_{o(1)}^2$ and $\alpha'_{\rm eff} = G_{o(1)}^2 \tilde{\alpha}'_{\rm eff}$. By taking further limits $G_o^2 \to \infty$, $\alpha'_{\rm eff} \to 0$ with $\alpha'_{\rm eff} G_o^2 =$ fixed in NCOS theory, which amounts to taking $G_{o(1)}^2 \to 0$, $\tilde{\alpha}'_{\rm eff} =$ fixed in OD1 theory both these theories reduce to Little String Theories [14]–[18]. Thus both these theories have (5+1) dimensional Lorentz invariance and reduce to Yang-Mills theory at low energies with coupling constant $g_{\rm YM}^2 = (2\pi)^3 G_o^2 \alpha'_{\rm eff} = (2\pi)^3 \tilde{\alpha}'_{\rm eff}$. In view of this observation, it was conjectured [4] that OD1 theory with coupling constant $G_{o(1)}$ and length scale $\tilde{\alpha}'_{\rm eff}$ may be identified with (5+1) dimensional NCOS with coupling constant $G_o = G_{o(1)}$ and $\alpha'_{\rm eff} = \tilde{\alpha}'_{\rm eff}/G_{o(1)}^2$. Since OD1 and (5+1) NCOS are S-dual to each other, so the above observation leads to the self-duality conjecture of both the theories.

By applying T-duality on (D1, NS5) solution along all directions transverse to D-strings and parallel to NS5-branes, it is easy to see that OD1 decoupling limit leads to OD5 decoupling limit [4]. In this case, an RR 6-form field strength attains the critical value and almost balances the tension of D5-branes on NS5-branes leading to OD5 theory. The (NS5, D5) bound state solution in the decoupling limit describes the supergravity dual [9, 12, 19] of OD5 theory.

In this paper we will consider an $SL(2, \mathbb{Z})$ invariant type IIB string theory vacuum of the form (F, D1, NS5, D5) which is a non-threshold bound states of (N, M) 5-branes with

[1]NCOS theory in (3+1) dimensions has been obtained in [7, 8].

the (N', M') strings of type IIB supergravity [20]. The reason for choosing this $\mathrm{SL}(2,\mathbb{Z})$ invariant bound state is that it will lead to the supergravity dual of various decoupled theories in (5+1) dimensions where the full $\mathrm{SL}(2,\mathbb{Z})$ transformation group of type IIB supergravity can be applied. This is in contrary to the special case bound states (F, D5) or (D1, NS5) where only the strong-weak duality behavior can be understood. It should be emphasized that NCOS, OD1 and OD5 theories obtained from this general supergravity configuration are different from those obtained from (F, D5), (D1, NS5) and (NS5, D5) separately in their field contents and therefore the decoupling limits are also modified. Also, the self-duality conjecture can be better addressed in the general $\mathrm{SL}(2,\mathbb{Z})$ invariant supergravity solution. The purpose of this paper is to illustrate the relationships between various decoupled theories under the general $\mathrm{SL}(2,\mathbb{Z})$ transformation of type IIB supergravity from the gravity point of view.

We will show here that starting from (F, D1, NS5, D5) bound state how all the three decoupling limits discussed above can be obtained from this single supergravity configuration and study their $\mathrm{SL}(2,\mathbb{Z})$ transformation properties.[2] Note that in this bound state strings lie along one of the spatial directions of the 5-branes and this solution preserves half of the space-time supersymmetries of the string theory. The supergravity solution corresponding to this bound state was constructed in [20]. It was mentioned there that the charges of the strings and 5-branes are not independent, but are given as $(N', M') = k(p, q)$ and $(N, M) = k'(-q, p)$ where (k, k') and (p, q) are two sets of relatively prime integers. So the number of NS5-branes are $|N| = k'q$ (henceforth we will call this simply as N) and the number of D5-branes are $M = k'p$ in this bound state. In the special case, when $q = 0$ and $p = 1$, we get (F, D5) bound state and when $p = 0$, $q = 1$, we get (D1, NS5) bound state.[3] We will first briefly review how the (5+1) dimensional NCOS theory and the OD1-theory are obtained as the decoupling limit on these two special bound states. Then we will show that under S-duality the full supergravity configuration of NCOS theory gets mapped to that of the OD1-theory, if we identify the parameters of these two theories in a particular way, showing that these two theories are related by strong-weak duality symmetry. Then we will take the full (F, D1, NS5, D5) bound state for the case where the asymptotic value of the axion (χ_0) present in this theory is rational, which can be put to zero and show how both NCOS-limit and OD1-limit can be taken from this configuration. Even though the field content of the supergravity configurations are different, we again find that under similar identification of parameters, S-dual NCOS theory gets mapped to OD1-theory. So, the same conclusion holds even for this more general NCOS theory and OD1 theory. One of the motivations for considering NCOS and OD1 limit for this more general solution was to see whether we can understand the self-duality conjecture[4] of both (5+1) dimensional NCOS and OD1-theory mentioned earlier. As we have already mentioned, the self duality

[2] $\mathrm{SL}(2,\mathbb{Z})$ transformation on the decoupling limit of (F, D1, D3) bound state has been studied in [21]–[24].

[3] It is also possible to obtain (NS5, D5) bound state of charges $(-q, p)$ for $k = 0$ and $k' = 1$ and (F, D1) bound state with four isometries (along x^2, x^3, x^4, x^5 directions) and charges (p, q) for $k' = 0$ and $k = 1$ from this general bound state. However, because of the charge relation given before, it is not possible to obtain (F, NS5) and (D1, D5) bound state from here [20].

[4] Indeed we have seen in [25] (see also [26, 27]) that the self-duality of OD3-theory can be shown from an $\mathrm{SL}(2,\mathbb{Z})$ invariant bound state configuration (NS5, D5, D3) of type IIB supergravity. We obtained an

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of either the (5+1) dimensional NCOS theory or OD1 theory means that we should be able to identify the OD1 theory with coupling constant $G_{o(1)}$ and length scale $\tilde{\alpha}'_{\text{eff}}$ with the NCOS theory with coupling constant $G_o = G_{o(1)}$ and length scale $\alpha'_{\text{eff}} = \tilde{\alpha}'_{\text{eff}}/G^2_{o(1)}$. If we can identify the corresponding supergravity duals of these two theories with the above identification, then it will lend support to the self-duality conjecture [4]. We will show that under appropriate identification of the parameters along with a condition the two supergravity configuration can indeed be identified. However, a closer look at some of the conditions required for the identification reveals that it can only be made in the range of the energy parameter in the NCOS theory where the dilaton blows up, i.e. where supergravity description breaks down. This may be an indication that the self-duality conjecture of either of these theories cannot be tested at the level of supergravity dual and some non perturbative techniques may be required.

We then study the general SL$(2,\mathbb{Z})$ transformation property of both NCOS theory and the OD1 theory for the SL$(2,\mathbb{Z})$ invariant configuration when the asymptotic value of the axion is irrational. Here we find that under a generic SL$(2,\mathbb{Z})$ transformation an NCOS theory always gives another NCOS theory with different values of the parameters. We give relations of the coupling constant and the length scales of the SL$(2,\mathbb{Z})$ transformed NCOS theory in terms of those of the original NCOS theory. However, the story is different for OD1-theory. For OD1 theory we find that even for the generic SL$(2,\mathbb{Z})$ transformation OD1-theory reduces to NCOS theory with rational χ_0. Since NCOS theory with rational χ_0 is mapped to OD1 theory with rational χ_0 under S-duality, so it shows that OD1 theory with irrational χ_0 is equivalent to the same theory with rational χ_0. So SL$(2,\mathbb{Z})$ transformation in this case does not give any new information.

Finally we also obtain an OD5-limit in the general SL$(2,\mathbb{Z})$ invariant supergravity configuration (F, D1, NS5, D5). Here we find that for rational χ_0, OD5 theory goes over to Little String Theory under S-duality of type IIB supergravity. However for irrational χ_0, the general SL$(2,\mathbb{Z})$ transformation gives another OD5 theory with different values of the parameters. The explicit relations between the parameters in the two OD5 theories are given.

The organization of the paper is as follows. In section 2, we give the supergravity configuration of (F, D1, NS5, D5) bound state. We briefly review the NCOS limit and the OD1-limit for the special cases of (F, D5) and (D1, NS5) configuration in section 3. Here we also discuss the S-duality between NCOS and OD1-theories. In section 4, both NCOS and OD1-limits are obtained for the general (F, D1, NS5, D5) bound state when χ_0 is rational. We discuss the S-duality as well as the self-duality of either NCOS or OD1 theory at the level of supergravity dual. The general SL$(2,\mathbb{Z})$ transformations when χ_0 is irrational is discussed in section 5. The OD5-limit and its SL$(2,\mathbb{Z})$ transformation for both the cases when χ_0 is rational as well as irrational is discussed in section 6. Our conclusion is presented in section 7.

OD3 decoupling limit for this solution and have shown that under S-duality, it gives rise to another OD3 theory with the same length scale as the original OD3-theory and the coupling constants are related as $\hat{G}_{o(3)} = 1/G_{o(3)}$.

2. The (F, D1, NS5, D5) bound state

The supergravity configuration of this bound state was constructed in [20]. Note that the metric in [20] was written in Einstein-frame which asymptotically becomes minkowskian. Also the solution was written for $\chi_0 = 0$. We here rewrite the solution with the metric in string frame such that the string frame metric becomes asymptotically minkowskian with an appropriate scaling of the coordinates. Also, we write the solution for $\chi_0 \neq 0$, by an $\mathrm{SL}(2,\mathbb{R})$ transformation on the solution of [20]. The configuration is,

$$
\begin{aligned}
ds^2 &= H'^{a1/2} H''^{1/2} \left[H^{-1}(-(dx^0)^2 + (dx^1)^2) + H'^{-1} \sum_{i=2}^{5} (dx^i)^2 + dr^2 + r^2 d\Omega_3^2 \right], \\
e^{\phi} &= g_s H^{-1/2} H'', \qquad \chi = \frac{\sin\varphi\cos\varphi(H-1)}{g_s H''} + \chi_0 = \frac{\tan\varphi(1 - H''^{-1})}{g_s} + \chi_0 , \\
F_{NS} &= \sin\varphi \sin\psi dH^{-1} \wedge dx^0 \wedge dx^1 + 2\cos\psi\cos\varphi\, Q_5\, \epsilon_3 , \\
F_{RR} &= -\left(\chi_0 - \frac{\cot\varphi}{g_s}\right) \sin\psi \sin\varphi\, dH^{-1} \wedge dx^0 \wedge dx^1 - 2\cos\psi\cos\varphi \left(\chi_0 + \frac{\tan\varphi}{g_s}\right) Q_5\, \epsilon_3 , \\
C_4 &= \frac{\tan\psi}{g_s}\left(1 - H'^{-1}\right) dx^2 \wedge dx^3 \wedge dx^4 \wedge dx^5 .
\end{aligned}
\tag{2.1}
$$

Note from (2.1) that the strings lie along x^1 direction and the 5-branes lie along x^1, x^2, x^3, x^4, x^5 directions. Also $r = \sqrt{x_6^2 + x_7^2 + x_8^2 + x_9^2}$, $d\Omega_3^2 = d\theta^2 + \sin^2\theta d\phi_1^2 + \sin^2\theta \sin^2\phi_1 d\phi_2^2$ is the line element of the unit 3-sphere transverse to the 5-branes and ϵ_3 is its volume form. $g_s = e^{\phi_0}$ is the string coupling constant, χ is the RR scalar (axion), F_{NS} and F_{RR} are respectively the NSNS and RR 3-form field strength. C_4 is the RR 4-form gauge field whose field strength is self dual. The Harmonic functions H, H', H'' are given as,

$$
\begin{aligned}
H &= 1 + \frac{Q_5}{r^2} , \\
H' &= 1 + \frac{\cos^2\psi Q_5}{r^2} , \\
H'' &= 1 + \frac{\cos^2\varphi Q_5}{r^2} ,
\end{aligned}
\tag{2.2}
$$

where the angles are defined as,

$$
\cos\varphi = \frac{q}{\sqrt{(p - \chi_0 q)^2 g_s^2 + q^2}} , \qquad \cos\psi = \frac{k'}{\sqrt{k^2 g_s^2 + k'^2}} . \tag{2.3}
$$

Here (p, q) and (k, k') are two sets of relatively prime integers which appear in the integral charges of (F, D1, NS5, D5) system as $(N', M', N, M) = (kp, kq, -k'q, k'p)$. Note that the number of NS5 branes is $|N| = k'q$ and D5 branes is $M = k'p$. The form of Q_5 in (2.2) is given as

$$
Q_5 = \left[(p - \chi_0 q)^2 g_s^2 + q^2\right]^{1/2} \left[k^2 g_s^2 + k'^2\right]^{1/2} \alpha' . \tag{2.4}
$$

Note that Q_5 can be expressed in terms of the angles as follows

$$
Q_5 = \frac{N\alpha'}{\cos\varphi\cos\psi} = \frac{(M - \chi_0 N)\, g_s \alpha'}{\sin\varphi\cos\psi} . \tag{2.5}
$$

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We will also use these forms of Q_5 while discussing OD1 and NCOS limits later. We note here that among the angles given in (2.3), $\cos\psi$ is $\mathrm{SL}(2,\mathbb{Z})$ invariant, but $\cos\varphi$ is not. Also Q_5 is $\mathrm{SL}(2,\mathbb{Z})$ invariant. Therefore while the harmonic functions H and H' are $\mathrm{SL}(2,\mathbb{Z})$ invariant H'' is not. The RR 4-form in (2.1) is $\mathrm{SL}(2,\mathbb{Z})$ invariant but the other fields change under $\mathrm{SL}(2,\mathbb{Z})$ transformation. These will be useful when we study the $\mathrm{SL}(2,\mathbb{Z})$ transformation of various decoupled theories.

3. NCOS and OD1 limit in the special cases and S-duality

We have already mentioned in the introduction that we can obtain (F, D5) bound state and (D1, NS5) bound state as special cases from the general bound state (F, D1, NS5, D5) given in the previous section. It is clear that when the integers $q = 0$, $p = 1$ and $\chi_0 = 0$, we get (F, D5) bound state with charges (k, k') and similarly when $q = 1, p = 0$ along with $\chi_0 = 0$ we get (D1, NS5) bound state with charges $(-k', k)$. We will review the NCOS and OD1 decoupling limits for (F, D5) and (D1, NS5) bound states repectively and show that the supergravity dual of NCOS theory gets mapped to that of OD1 theory by S-duality. This has already been discussed in [11], but we will show the full mapping including the NSNS and RR gauge fields.

3.1 (F, D5) solution and NCOS-limit

When $q = 0$, $p = 1$ and $\chi_0 = 0$ we find from (2.3) that $\cos\varphi = 0$. Therefore, $H'' = 1$. In this case (F, D1, NS5, D5) configuration in (2.1) reduces to (F, D5) solution and is given as,

$$\begin{aligned}
ds^2 &= H'^{1/2}\left[H^{-1}(-(dx^0)^2 + (dx^1)^2) + H'^{-1}\sum_{i=2}^{5}(dx^i)^2 + dr^2 + r^2 d\Omega_3^2\right],\\
e^{\phi} &= g_s H^{-1/2}\,, \qquad \chi = 0\,,\\
F_{NS} &= \sin\psi dH^{-1}\wedge dx^0\wedge dx^1\,,\\
F_{RR} &= -\frac{2\cos\psi}{g_s}Q_5\,\epsilon_3\,,\\
C_4 &= \frac{\tan\psi}{g_s}\left(1 - H'^{-1}\right) dx^2\wedge dx^3\wedge dx^4\wedge dx^5\,.
\end{aligned}\tag{3.1}$$

From the second expression of (2.5) we find the form of Q_5 as

$$Q_5 = \frac{M g_s \alpha'}{\cos\psi}\tag{3.2}$$

and so the harmonic functions have the forms (see eq. (2.2))

$$\begin{aligned}
H &= 1 + \frac{M g_s\alpha'}{\cos\psi r^2}\,,\\
H' &= 1 + \frac{\cos\psi M g_s\alpha'}{r^2}\,,
\end{aligned}\tag{3.3}$$

where M is the number of D5-branes. The NCOS decoupling limit [4] can be summarized as follows. We define a positive, dimensionless scaling parameter ϵ and take $\cos\psi = \epsilon \to 0$, keeping the following quantities fixed,

$$\alpha'_{\rm eff} = \frac{\alpha'}{\epsilon}, \qquad G_o^2 = \epsilon g_s, \qquad u = \frac{r}{\sqrt{\epsilon}\alpha'_{\rm eff}}\,. \tag{3.4}$$

In the above G_o^2 is the coupling constant, $\alpha'_{\rm eff}$ is the length scale and u is the energy parameter in the NCOS theory. Under this limit, the harmonic functions in (3.3) reduce to

$$H = \frac{1}{a^2\epsilon^2 u^2}\,, \qquad H' = \frac{h'}{a^2u^2}\,, \tag{3.5}$$

where we have defined $a^2 = \alpha'_{\rm eff}/MG_o^2$ and $h' = 1 + a^2u^2$. Then the supergravity configuration in (3.1) take the forms

$$\begin{aligned}
ds^2 &= \epsilon h'^{1/2} au\left[-(d\tilde{x}^0)^2 + (d\tilde{x}^1)^2 + h'^{-1}\sum_{i=2}^{5}(d\tilde{x}^i)^2 + \frac{MG_o^2\alpha'_{\rm eff}}{u^2}\left(du^2 + u^2 d\Omega_3^2\right)\right],\\
e^{\phi} &= G_o^2 au\,,\\
B_{01} &= \epsilon a^2u^2\,,\\
F_{RR} &= -2\epsilon M\alpha'_{\rm eff}\epsilon_3\,,\\
C_{2345} &= \epsilon^2/(G_o^2h')\,.
\end{aligned} \tag{3.6}$$

In the above we have defined the fixed coordinates as

$$\tilde{x}^{0,1} = \sqrt{\epsilon}x^{0,1}\,, \qquad \tilde{x}^{2,\ldots,5} = \frac{1}{\sqrt{\epsilon}}x^{2,\ldots,5}\,. \tag{3.7}$$

Note that this is precisely the (5+1) dimensional NCOS limit given in [4]. Also B_{01} in (3.6) is the NSNS 2-form potential in component form and C_{2345} is the RR 4-form potential in component form. (3.6) describes the supergravity dual of (5+1) dimensional NCOS theory. The gravity dual description is valid as long as $e^{\phi} \ll 1$ and the curvature measured in units of α' remains small.

3.2 (D1, NS5) solution and OD1 limit

The (D1, NS5) solution can be obtained from (2.1) when $q = 1$, $p = 0$ and also $\chi_0 = 0$. In this case $\cos\varphi = 1$ (from (2.3)) and therefore from (2.2) harmonic functions $H = H''$. This solution now takes the form,

$$\begin{aligned}
d\bar{s}^2 &= H'^{1/2}H^{1/2}\left[H^{-1}(-(dx^0)^2 + (dx^1)^2) + H'^{-1}\sum_{i=2}^{5}(dx^i)^2 + dr^2 + r^2d\Omega_3^2\right],\\
e^{\bar{\phi}} &= g_sH^{1/2}\,, \qquad \bar{\chi} = 0\,,\\
\bar{F}_{NS} &= 2\cos\psi Q_5\epsilon_3\,,\\
\bar{F}_{RR} &= \frac{\sin\psi}{g_s}dH^{-1}\wedge dx^0\wedge dx^1\,,\\
\bar{C}_4 &= \frac{\tan\psi}{g_s}\left(1 - H'^{-1}\right)dx^2\wedge dx^3\wedge dx^4\wedge dx^5\,.
\end{aligned} \tag{3.8}$$

To avoid notational confusion we have denoted the fields of (D1, NS5) solution with a 'bar'. Now from the first expression of (2.5) we have the form of Q_5 as

$$Q_5 = \frac{N\alpha'}{\cos\psi} \tag{3.9}$$

and therefore the harmonic functions in (2.2) are given as

$$H = 1 + \frac{N\alpha'}{\cos\psi r^2}\,, \qquad H' = 1 + \frac{\cos\psi N\alpha'}{r^2}\,, \tag{3.10}$$

where N is the number of NS5-branes. The OD1 decoupling limit is obtained by defining the positive scaling parameter ϵ and taking $\cos\psi = \epsilon \to 0$, keeping the following quantities fixed

$$\tilde{\alpha}'_{\rm eff} = \frac{\tilde{\alpha}'}{\epsilon}\,, \qquad G^2_{o(1)} = \frac{g_s}{\epsilon}\,, \qquad \tilde{u} = \frac{r}{\epsilon\tilde{\alpha}'_{\rm eff}}\,, \tag{3.11}$$

where $\tilde{\alpha}' = \alpha'/G^2_{o(1)}$. In the above $\tilde{\alpha}'_{\rm eff}$ is the length scale, $G^2_{o(1)}$ is the coupling constant and $\tilde{u}$ is the energy parameter in the OD1 theory. In this limit the harmonic functions in (3.10) are given as

$$H = \frac{1}{c^2\epsilon^2\tilde{u}^2}\,, \qquad H' = \frac{f'}{c^2\tilde{u}^2}\,, \tag{3.12}$$

where we have defined $c^2 = \tilde{\alpha}'_{\rm eff}/(NG^2_{o(1)})$ and $f' = 1+c^2\tilde{u}^2$. The supergravity configuration in (3.8) then takes the form,

$$\begin{aligned}
d\bar{s}^2 &= \epsilon G^2_{o(1)} f'^{1/2}\left[-(d\bar{x}^0)^2 + (d\bar{x}^1)^2 + f'^{-1}\sum_{i=2}^{5}(d\bar{x}^i)^2 + \frac{N\tilde{\alpha}'_{\rm eff}}{\tilde{u}^2}\left(d\tilde{u}^2 + \tilde{u}^2 d\Omega_3^2\right)\right],\\
e^{\bar{\phi}} &= \frac{G^2_{o(1)}}{c\tilde{u}}\,,\\
\bar{F}_{NS} &= 2\epsilon N G^2_{o(1)}\tilde{\alpha}'_{\rm eff}\epsilon_3\\
\bar{C}_{01} &= \epsilon c^2\tilde{u}^2\,,\\
\bar{C}_{2345} &= \epsilon^2 G^2_{o(1)}/f'\,.
\end{aligned} \tag{3.13}$$

In the above we have defined the fixed coordinates as,

$$\bar{x}^{0,1} = \frac{x^{0,1}}{G_{o(1)}}\,, \qquad \bar{x}^{2,\ldots,5} = \frac{1}{\epsilon G_{o(1)}}x^{2,\ldots,5}\,. \tag{3.14}$$

Here $\bar{C}_{01}$ is the RR 2-form potential in the component form and $\bar{C}_{2345}$ is the RR 4-form potential in component form. Eq. (3.13) describes the supergravity dual of OD1 theory.

3.3 S-duality between NCOS and OD1 theory

In this subsection we will show that under the S-duality of type IIB supergravity, the gravity dual configuration of NCOS theory gets mapped to those of OD1 theory. Note that the S-dual configuration of NCOS theory would be given as

$$\begin{aligned}
d\hat{s}^2 &= e^{-\phi} ds^2 \,, \\
e^{\hat{\phi}} &= e^{-\phi} \,, \\
\hat{F}_{NS} &= -F_{RR} \,, \\
\hat{C}_{01} &= B_{01} \,, \\
\hat{C}_{2345} &= C_{2345} \,.
\end{aligned} \tag{3.15}$$

Using these we get from (3.6) the S-dual supergravity configuration of NCOS theory as,

$$\begin{aligned}
d\hat{s}^2 &= \frac{\epsilon h'^{1/2}}{G_o^2}\left[-(d\tilde{x}^0)^2 + (d\tilde{x}^1)^2 + h'^{-1}\sum_{i=2}^{5}(d\tilde{x}^i)^2 + \frac{MG_o^2\alpha'_{\rm eff}}{u^2}\left(du^2 + u^2 d\Omega_3^2\right)\right], \\
e^{\hat{\phi}} &= \frac{1}{G_o^2 au} \,, \\
\hat{F}_{NS} &= 2\epsilon M\alpha'_{\rm eff}\epsilon_3 \,, \\
\hat{C}_{01} &= \epsilon a^2 u^2 \,, \\
\hat{C}_{2345} &= \epsilon^2/(G_o^2 h') \,.
\end{aligned} \tag{3.16}$$

Here $\tilde{x}^{0,1}$ and $\tilde{x}^{2,\ldots,5}$ are the same as given in (3.7). Comparing (3.16) with the field in OD1 theory given in (3.13) we find that they match exactly with the following identification

$$\begin{aligned}
&h' = f' \,, \qquad G_o^2 = \frac{1}{G_{o(1)}^2} \,, \qquad \alpha'_{\rm eff} = G_{o(1)}^2\tilde{\alpha}'_{\rm eff}, \qquad M = N \,, \\
\text{and} \quad &\tilde{x}^{0,1} = \bar{x}^{0,1} \,, \qquad \tilde{x}^{2,\ldots,5} = \bar{x}^{2,\ldots,5} \,.
\end{aligned} \tag{3.17}$$

The first condition in (3.17) implies

$$a^2u^2 = c^2\tilde{u}^2 \quad \Rightarrow \quad u = \frac{\tilde{u}}{G_{o(1)}^3} \,. \tag{3.18}$$

This gives a relation between the energy parameters in NCOS theory and OD1 theory. Thus we find that the coupling constant and the length scales in these two theories are related as given in (3.17) showing that these two theories are S-dual to each other. From the coordinate relation given in (3.17), it is clear that the coordinates of the gravity dual configurations of NCOS theory ($x^{0,1,\ldots,5}$(NCOS)) and OD1 theory ($x^{0,1,\ldots,5}$(OD1)) have a relative scaling of the form $x^{0,1,\ldots,5}(\text{NCOS}) = 1/(G_{o(1)}\sqrt{\epsilon})x^{0,1,\ldots,5}(\text{OD1})$. Similarly, the radial coordinates in these two theories also have a relative scaling $r(\text{NCOS}) = 1/(G_{o(1)}\sqrt{\epsilon})r(\text{OD1})$ as can be seen from (3.4) and (3.11) if the parameters in these two theories are related as in (3.17) and (3.18). The reason for this can be traced back as follows. Note that the metric for both (F, D5) and (D1, NS5) supergravity configuration given respectively in (3.1) and (3.8) are asymptotically minkowskian and we have taken decoupling limits on them. However, when we take S-duality on the gravity dual of NCOS in (3.15), the S-dual metric ($d\hat{s}^2$) is not asymptotically Minkowskian, since, as usual, we have absorbed a factor of g_s in ds^2 in the r.h.s. of the metric expression in (3.15). So, to compensate this effect we have to multiply a $g_s^{-1/2} = \sqrt{\epsilon}/G_o (= \sqrt{\epsilon} G_{o(1)})$ on the NCOS coordinate while the OD1 coordinates remain the same. We note by comparing the gravity dual configurations of NCOS theory in (3.6) and OD1 theory in (3.13) that they cannot be identified with the parametric relations $G_o = G_{o(1)}$ and $\alpha'_{\text{eff}} = \tilde{\alpha}'_{\text{eff}}/G^2_{o(1)}$ and so the self-duality conjecture of (5+1) dimensional NCOS and OD1 cannot be tested at the level of gravity dual. We will mention more about it in the context of full (F, D1, NS5, D5) solution in the next section.

4. NCOS and OD1 limit for (F, D1, NS5, D5) bound state with rational χ_0 and S-duality

In this section we consider the full (F, D1, NS5, D5) supergravity configuration with rational χ_0. When χ_0 is rational we can always make an $\mathrm{SL}(2,\mathbb{Z})$ transformation such that χ_0 vanishes. Therefore, we will take the supergravity configuration (2.1) with $\chi_0 = 0$. We will obtain both the NCOS limit and OD1 limit for this configuration discussed in the previous section. We will show as in the previous section that the supergravity dual of NCOS and OD1 get mapped to each other under S-duality. We will also comment on the self-duality of (5+1) dimensional NCOS theory.

4.1 (F, D1, NS5, D5) solution and NCOS limit

The NCOS limit for the (F, D5) solution is given in (3.4). In that case $\cos\varphi = 0$, but here we will take $\cos\varphi$ to scale as

$$\cos\varphi = l\epsilon\,, \tag{4.1}$$

where l is a finite parameter such that $l\epsilon \to 0$, but $|l|$ obviously has to be less than $1/\epsilon$. So eq. (3.4) along with (4.1) define the NCOS limit in this case. The harmonic functions (2.2) in this case take the forms,

$$\begin{aligned} H &= 1 + \frac{M g_s \alpha'}{\cos\psi \sin\varphi r^2}\,, \\ H' &= 1 + \frac{\cos\psi M g_s \alpha'}{\sin\varphi r^2}\,, \\ H'' &= 1 + \frac{\cos^2\varphi M g_s \alpha'}{\cos\psi \sin\varphi r^2}\,, \end{aligned} \tag{4.2}$$

where we have used the form of Q_5 in (2.5). Here $M = k'p$ denotes the number of D5-branes. Now in the decoupling limit (3.4) and (4.1) the harmonic functions reduce to

$$H = \frac{1}{a^2\epsilon^2 u^2}\,, \qquad H' = \frac{h'}{a^2 u^2}\,, \qquad H'' = \frac{h''}{\tilde{a}^2 u^2}\,. \tag{4.3}$$

In the above $a^2 = \alpha'_{\text{eff}}/(MG_o^2)$ as in (3.5) and $h' = 1 + a^2u^2$, but $\tilde{a}^2 = \alpha'_{\text{eff}}/(MG_o^2 l^2)$ and $h'' = 1 + \tilde{a}^2 u^2$. We thus have $a/\tilde{a} = l$ and $a\tilde{a} = \alpha'_{\text{eff}}/(MG_o^2 l)$. Using the decoupling limit and eq. (4.3), we find that the full supergravity configuration (eq. (2.1) with $\chi_0 = 0$)

reduce to,

$$
\begin{aligned}
ds^2 &= \epsilon l h'^{1/2} h''^{1/2}\left[-(d\tilde{x}^0)^2 + (d\tilde{x}^1)^2 + h'^{-1}\sum_{i=2}^{5}(d\tilde{x}^i)^2 + \frac{MG_o^2\alpha'_{\rm eff}}{u^2}\left(du^2 + u^2 d\Omega_3^2\right)\right],\\
e^{\phi} &= G_o^2 h'' \frac{l^2}{au} = \frac{G_o^2 h'' l}{\tilde{a}u}, \qquad \chi = \frac{1}{l h'' G_o^2},\\
B_{01} &= \epsilon a^2 u^2, \qquad F_{NS} = 2\epsilon l M G_o^2 \alpha'_{\rm eff}\epsilon_3,\\
C_{01} &= \epsilon^3 \frac{l}{G_o^2} a^2 u^2, \qquad F_{RR} = -2\epsilon M \alpha'_{\rm eff}\epsilon_3,\\
C_{2345} &= \epsilon^2/(G_o^2 h').
\end{aligned}
\tag{4.4}
$$

In the above the fixed coordinates $\tilde{x}^{0,1,\ldots,5}$ are again as given before in (3.7). Note from eq. (2.1) that NSNS and RR 3-form field strengths have two parts each, so in (4.4) B_{01} and C_{01} denote respectively the 01 components of NSNS and RR 2-form potentials. The other parts F_{NS} and F_{RR} are kept as it is. C_{2345} is the 4-form potential in component form. Eq. (4.4) represents the gravity dual description of NCOS theory.

4.2 (F, D1, NS5, D5) solution and OD1 limit

The OD1 limit for the (D1, NS5) solution is given in (3.11). In that case the angle $\cos\varphi = 1$ i.e. $\sin\varphi = 0$, but here we will take $\sin\varphi$ to scale as,

$$\sin\varphi = \tilde{l}\epsilon\,, \tag{4.5}$$

where $\tilde{l}$ is another finite parameter such that $\tilde{l}\epsilon \to 0$, with $|\tilde{l}| < 1/\epsilon$. So, eq. (3.11) along with (4.5) define the OD1-limit in this case. The harmonic functions (2.2) for this case take the forms,

$$
\begin{aligned}
H &= 1 + \frac{N\alpha'}{\cos\psi\cos\varphi r^2},\\
H' &= 1 + \frac{\cos\psi N\alpha'}{\cos\varphi r^2},\\
H'' &= 1 + \frac{\cos\varphi N\alpha'}{\cos\psi r^2},
\end{aligned}
\tag{4.6}
$$

where we have used the form of Q_5 in eq (2.5). Here $N = k'q$ represents the number of NS5-branes. In the OD1 decoupling limit given by eq. (3.11) and (4.5), the harmonic functions in (4.6) reduce to

$$H = \frac{1}{c^2\epsilon^2\tilde{u}^2}, \qquad H' = \frac{f'}{c^2\tilde{u}^2}, \qquad H'' = \frac{1}{c^2\epsilon^2\tilde{u}^2}. \tag{4.7}$$

In the above $c^2 = \tilde{\alpha}'_{\rm eff}/(NG_{o(1)}^2)$ as given before and $f' = 1 + c^2\tilde{u}^2$. Using this, the full supergravity configuration (eq. (2.1) with $\chi_0 = 0$) takes the form,

$$
\begin{aligned}
d\bar{s}^2 &= \epsilon f'^{1/2} G^2_{o(1)} \left[-(d\bar{x}^0)^2 + (d\bar{x}^1)^2 + f'^{-1} \sum_{i=2}^{5} (d\bar{x}^i)^2 + \frac{N\tilde{\alpha}'_{\rm eff}}{\tilde{u}^2} \left(d\tilde{u}^2 + \tilde{u}^2 d\Omega_3^2 \right) \right], \\
e^{\bar{\phi}} &= \frac{G^2_{o(1)}}{c\tilde{u}}, \qquad \bar{\chi} = \frac{\tilde{l}}{G^2_{o(1)}}, \\
\bar{B}_{01} &= \epsilon^3 \tilde{l} G^2_{o(1)} c^2 \tilde{u}^2, \qquad \bar{F}_{NS} = 2\epsilon N G^2_{o(1)} \tilde{\alpha}'_{\rm eff} \epsilon_3, \\
\bar{C}_{01} &= \epsilon c^2 \tilde{u}^2, \qquad \bar{F}_{RR} = -2\epsilon \tilde{l} N \tilde{\alpha}'_{\rm eff} \epsilon_3, \\
\bar{C}_{2345} &= \epsilon^2 G^2_{o(1)}/f'.
\end{aligned} \tag{4.8}
$$

Here the fixed coordinates, $\bar{x}^{0,1,\ldots,5}$ are the same as defined before in (3.14). So eq. (4.8) represents the gravity dual description of OD1 theory.

4.3 S-duality between NCOS and OD1 theory in general case

We have obtained the gravity dual configurations of NCOS and OD1 theory from the decoupling limits on the same supergravity solution (F, D1, NS5, D5) in eqs. (4.4) and (4.8). In this subsection we will show that these two theories are related by S-duality even for these more general NCOS and OD1 theories. The S-dual configuration of NCOS theory would be given as

$$
\begin{aligned}
d\hat{s}^2 &= |\lambda| ds^2, \\
e^{\hat{\phi}} &= |\lambda|^2 e^{\phi}, \qquad \hat{\chi} = -\frac{\chi}{|\lambda|^2}, \\
\hat{B}_{01} &= -C_{01}, \qquad \hat{F}_{NS} = -F_{RR}, \\
\hat{C}_{01} &= B_{01}, \qquad \hat{F}_{RR} = F_{NS}, \\
\hat{C}_{2345} &= C_{2345},
\end{aligned} \tag{4.9}
$$

where $|\lambda| = \sqrt{\chi^2 + e^{-2\phi}}$. As before here also we have absorbed a factor of $|\lambda_0|^{-1}$ in ds^2 on the r.h.s. of the metric expression in (4.9). The value of $|\lambda|$ can be calculated from (4.4) and has the form $|\lambda| = 1/(lG_o^2 h''^{1/2})$.

So, using (4.9) and (4.4), we find the S-dual configuration of NCOS theory as,

$$
\begin{aligned}
d\hat{s}^2 &= \frac{\epsilon h'^{1/2}}{G_o^2} \left[-(d\tilde{x}^0)^2 + (d\tilde{x}^1)^2 + h'^{-1} \sum_{i=2}^{5} (d\tilde{x}^i)^2 + \frac{MG_o^2 \alpha'_{\rm eff}}{u^2} \left(du^2 + u^2 d\Omega_3^2 \right) \right], \\
e^{\hat{\phi}} &= \frac{1}{G_o^2 a u}, \qquad \hat{\chi} = -lG_o^2, \\
\hat{B}_{01} &= -\epsilon^3 \frac{l}{G_o^2} a^2 u^2, \qquad \hat{F}_{NS} = 2\epsilon M \alpha'_{\rm eff} \epsilon_3, \\
\hat{C}_{01} &= \epsilon a^2 u^2, \qquad \hat{F}_{RR} = 2\epsilon l M G_o^2 \alpha'_{\rm eff} \epsilon_3, \\
\hat{C}_{2345} &= \frac{\epsilon^2}{G_o^2 h'}.
\end{aligned} \tag{4.10}
$$

By comparing the S-dual NCOS configuration (4.10) with the OD1 configuration given in (4.8), we find that all the fields indeed match exactly with the same identification of parameters (3.17) and (3.18) along with an extra condition

$$l = -\tilde{l}\,. \tag{4.11}$$

This therefore shows that under S-duality the gravity dual configuration of NCOS theory gets mapped to that of OD1 theory. So, indeed these two theories are related by strong-weak duality symmetry.

4.4 On self-duality of (5+1) dimensional NCOS or OD1

We have already shown in the previous subsection that for the general case NCOS theory is S-dual to OD1 theory with the parametric relations $G_o^2 = 1/G_{o(1)}^2$ and $\alpha'_{\rm eff} = G_{o(1)}^2 \tilde{\alpha}'_{\rm eff}$. So the strongly coupled NCOS theory gets mapped to the weakly coupled region of OD1 theory and vice-versa. However, in order to show the self-duality of either the (5+1) NCOS theory or OD1 theory we must show that these two theories may be identified with the parametric relations $G_o = G_{o(1)}$ and $\alpha'_{\rm eff} = \tilde{\alpha}'_{\rm eff}/G_{o(1)}^2$. In other words, the strong coupling region of NCOS (OD1) theory must get mapped to the strong coupling region of OD1 (NCOS) theory or the weak coupling region of one theory must map to the weak coupling region of other theory. We will try to see whether at the level of supergravity dual we can make this identification. However, note that the same OD1 limit (3.11) and (4.5) will not do the job and we need to modify it by replacing $G_{o(1)}^2 \to 1/G_{o(1)}^2$. So, the OD1 limit we take is the following.

$$\epsilon \to 0\,, \qquad \cos\psi = \epsilon \to 0\,, \qquad \sin\varphi = \tilde{l}\epsilon \to 0\,, \tag{4.12}$$

keeping the following quantities fixed,

$$\tilde{\alpha}'_{\rm eff} = \frac{\tilde{\alpha}'}{\epsilon}\,, \qquad G_{o(1)}^2 = \frac{\epsilon}{g_s}\,, \qquad \tilde{u} = \frac{r}{\epsilon \tilde{\alpha}'_{\rm eff}}\,, \tag{4.13}$$

where $\tilde{\alpha}' = G_{o(1)}^2 \alpha'$. The harmonic functions (4.6) in this limit take the forms,

$$H = \frac{1}{b^2\epsilon^2\tilde{u}^2}\,, \qquad H' = \frac{g'}{b^2\tilde{u}^2}\,, \qquad H'' = \frac{1}{b^2\epsilon^2\tilde{u}^2}\,, \tag{4.14}$$

where $b^2 = G_{o(1)}^2\tilde{\alpha}'_{\rm eff}/N$ and $g' = 1 + b^2\tilde{u}^2$. The supergravity configuration is therefore given as,

$$\begin{aligned}
d\bar{s}^2 &= \frac{\epsilon g'^{1/2}}{G_{o(1)}^2}\left[-(d\bar{x}^0)^2 + (d\bar{x}^1)^2 + g'^{-1}\sum_{i=2}^{5}(d\bar{x}^i)^2 + \frac{N\tilde{\alpha}'_{\rm eff}}{\tilde{u}^2}\left(d\tilde{u}^2 + \tilde{u}^2 d\Omega_3^2\right)\right],\\
e^{\bar{\phi}} &= \frac{1}{G_{o(1)}^2 b\tilde{u}}\,, \qquad \bar{\chi} = \tilde{l}G_{o(1)}^2\,,\\
\bar{B}_{01} &= \frac{\epsilon^3\tilde{l}}{G_{o(1)}^2}b^2\tilde{u}^2\,, \qquad \bar{F}_{NS} = \frac{2N\epsilon\tilde{\alpha}'_{\rm eff}}{G_{o(1)}^2}\epsilon_3\,,\\
\bar{C}_{01} &= \epsilon b^2\tilde{u}^2\,, \qquad \bar{F}_{RR} = -2N\epsilon\tilde{l}\tilde{\alpha}'_{\rm eff}\epsilon_3\,,\\
\bar{C}_{2345} &= \frac{\epsilon^2}{G_{o(1)}^2 g'}\,,
\end{aligned} \tag{4.15}$$

where the fixed coordinates are defined as,

$$\bar{x}^{0,1} = G_{o(1)} x^{0,1}\,, \qquad \text{and} \qquad \bar{x}^{2,\ldots,5} = \frac{G_{o(1)}}{\epsilon} x^{2,\ldots,5}\,. \tag{4.16}$$

By comparing (4.15) with the NCOS supergravity configuration (4.4) we find that the fields may be identified as the following,

$$\begin{aligned} d\bar{s}^2 &= ds^2\,, \qquad e^{\bar{\phi}} = e^{\phi}\,, \qquad \bar{\chi} = -\chi\,, \qquad \bar{B}_{01} = -C_{01}\,, \qquad \bar{F}_{NS} = -F_{RR} \\ \bar{C}_{01} &= B_{01}\,, \qquad \bar{F}_{RR} = F_{NS}\,, \qquad \text{and} \qquad \bar{C}_{2345} = C_{2345}\,, \end{aligned} \tag{4.17}$$

if we impose the following conditions on the parameters of the two theories,

$$\begin{aligned} l h''^{1/2} &= \frac{1}{G_o^2}\,, \qquad h' = g'\,, \qquad G_o^2 = G_{o(1)}^2\,, \qquad \alpha'_{\text{eff}} = \frac{\tilde{\alpha}'_{\text{eff}}}{G_{o(1)}^2}\,, \\ M &= N\,, \qquad \text{and} \qquad l = -\tilde{l}\,. \end{aligned} \tag{4.18}$$

Note that the second condition in (4.18) gives a relation between the energy parameters of NCOS theory (u) and OD1 theory ($\tilde{u}$) as,

$$u = \tilde{u} G_{o(1)}^3\,, \qquad \text{or} \qquad \tilde{u} = \frac{u}{G_o^3}\,, \tag{4.19}$$

very similar to the relation we have already found in (3.18). From (4.17) and (4.18), it might seem that we have been able to identify NCOS theory woth OD1 theory implying that we have been able to show the self-duality conjecture at the level of supergravity dual. However, this is not true. In fact if we look closely to the first condition in (4.18), we find that it implies[5]

$$h'' \approx 1\,, \qquad \text{and} \qquad l = \frac{1}{G_o^2}\,. \tag{4.20}$$

$h'' \approx 1$ implies $\tilde{a}^2 u^2 \ll 1$ or in other words, the above identification can be made only if the energy parameter in the NCOS theory satisfies

$$u^2 \ll \frac{M}{\alpha'_{\text{eff}} G_o^2}\,. \tag{4.21}$$

Note that this does not necessarily imply that $h' \approx 1$ also since $a^2 = \alpha'_{\text{eff}}/(M G_o^2)$. But from the expression of e^{ϕ} in (4.4) i.e. for NCOS theory, we find that precisely in the energy region (4.21), it blows up indicating that the supergravity description breaks down.[6] This clearly shows that the identification of NCOS and OD1 theory can not be made at the

[5] If we do not separate the condition as given below in eq. (4.20), then this implies that the field identification (4.17) can be made only at a single value of the energy given by $u^2 = M/\alpha'_{\text{eff}} G_o^2 - M G_o^2 l^2/\alpha'_{\text{eff}}$. Even in this case the effective coupling e^{ϕ} does not remain small since $e^{\phi} \ll 1 \Rightarrow u^2 \gg M/\alpha'_{\text{eff}} G_o^2$. So supergravity description can not be trusted.

[6] The condition $h'' \approx 1$, $h' \not\approx 1$ can be satisfied if $G_o^2 \ll 1$ i.e. in the weak coupling region of NCOS (or OD1) theory. But note that for $G_o^2 \gg 1$, h'' and h' get interchanged and we have $h' \approx 1$, $h'' \not\approx 1$. Thus we still get the mapping of NCOS and OD1 fields with h'' and h' interchanged and the same conclusion holds for this case.

level of supergravity dual and therefore in order to test the self-duality conjecture some non-perturbative techniques may be required. However, we would like to remark that since we have been able to map the fields of NCOS theory with those of OD1 theory as expected from various low energy arguments mentioned in the introduction, it might be possible that the supergravity description remains valid even in the strong coupling region (where $e^{\phi} \gg 1$) due to some underlying non-renormalization effect. But we can not justify the validity of such remark any further at this point.

5. (F, D1, NS5, D5) solution with irrational χ_0, NCOS, OD1 limits and $\mathrm{SL}(2,\mathbb{Z})$ duality

In this section we will study the (F, D1, NS5, D5) supergravity configuration for irrational χ_0 and obtain both NCOS and OD1 limits. Since type IIB string theory is believed to possess an $\mathrm{SL}(2,\mathbb{Z})$ invariance and (F, D1, NS5, D5) state is $\mathrm{SL}(2,\mathbb{Z})$ invariant, we also discuss the $\mathrm{SL}(2,\mathbb{Z})$ transformation of both these theories.

Under an $\mathrm{SL}(2,\mathbb{Z})$ transformation by the matrix $\left(\begin{smallmatrix} v & w \\ r & s \end{smallmatrix}\right)$, where v, w, r, s are integers with $vs - rw = 1$, the various fields of type IIB supergravity transform as,

$$\begin{aligned} &g^E_{\mu\nu} \to g^E_{\mu\nu}, \qquad \tau \to \frac{v\tau + w}{r\tau + s}, \qquad \begin{pmatrix} F_{NS} \\ F_{RR} \end{pmatrix} \to \begin{pmatrix} s & -r \\ -w & v \end{pmatrix} \begin{pmatrix} F_{NS} \\ F_{RR} \end{pmatrix} \\ &C_4 \to C_4\,, \end{aligned} \tag{5.1}$$

where $g^E_{\mu\nu}$ is the Einstein metric and $\tau = \chi + ie^{-\phi}$. The explicit transformation of the dilaton and the axion are

$$\begin{aligned} e^{\hat{\phi}} &= |r\tau + s|^2 e^{\phi} = |\lambda|^2 e^{\phi} \\ \hat{\chi} &= \frac{(v\chi + w)(r\chi + s) + vre^{-2\phi}}{|r\tau + s|^2} \end{aligned} \tag{5.2}$$

and the string frame metric $g_{\mu\nu} = e^{\phi/2} g^E_{\mu\nu}$ transforms as,

$$d\hat{s}^2 = |\lambda| ds^2\,, \tag{5.3}$$

where we have defined $|\lambda| = |r\tau + s|$. If we demand that the transformed metric be asymptotically minkowskian then

$$d\hat{s}^2 = \frac{|\lambda|}{|\lambda_0|} ds^2\,. \tag{5.4}$$

In the above $|\lambda_0| = |r\tau_0 + s| = \left[(r\chi_0 + s)^2 + r^2 e^{-2\phi_0}\right]^{1/2}$, with $e^{\phi_0} = g_s$, the closed string coupling and χ_0, the asymptotic value of the axion. We will use these transformations to study the $\mathrm{SL}(2,\mathbb{Z})$ duality of the decoupled theories.

5.1 NCOS limit and $\mathrm{SL}(2,\mathbb{Z})$ duality

The supergravity solution is given in (2.1), where the harmonic functions are now of the forms,

$$\begin{aligned} H &= 1 + \frac{(M-\chi_0 N) g_s \alpha'}{\cos\psi \sin\varphi r^2}\,, \\ H' &= 1 + \frac{(M-\chi_0 N) g_s \alpha' \cos\psi}{\sin\varphi r^2}\,, \\ H'' &= 1 + \frac{(M-\chi_0 N) g_s \alpha' \cos^2\varphi}{\cos\psi \sin\varphi r^2}\,. \end{aligned} \tag{5.5}$$

In the NCOS decoupling limit (3.4) and (4.1), they reduce to

$$H = \frac{1}{a^2\epsilon^2 u^2}\,, \qquad H' = \frac{h'}{a^2 u^2}\,, \qquad H'' = \frac{h''}{\tilde{a}^2 u^2}\,, \tag{5.6}$$

where $a^2 = \alpha'_{\rm eff}/((M-\chi_0 N)G_o^2)$, $h' = 1 + a^2 u^2$ and $\tilde{a}^2 = \alpha'_{\rm eff}/((M-\chi_0 N)G_o^2 l^2)$, $h'' = 1 + \tilde{a}^2 u^2$. Note that the parameters a and $\tilde{a}$ here have different values from the previous sections. Using (5.1) and the decoupling limit (3.4) and (4.1) we get the metric, dilaton and axion from eq. (2.1) in the following form[7]

$$\begin{aligned} ds^2 &= \epsilon l h'^{1/2} h''^{1/2} \times \\ &\quad\times \left[-(d\tilde{x}^0)^2 + (d\tilde{x}^1)^2 + h'^{-1} \sum_{i=2}^{5} (d\tilde{x}^i)^2 + \frac{(M-\chi_0 N)\alpha'_{\rm eff}}{u^2}(du^2 + u^2 d\Omega_3^2) \right] \\ e^\phi &= \frac{G_o^2 l^2 h''}{au}\,, \qquad \chi = \frac{1}{h'' G_o^2 l} + \chi_0\,. \end{aligned} \tag{5.7}$$

where $\tilde{x}^{0,1,\ldots,5}$ are the same as given in (3.7). This is the gravity dual of NCOS theory when χ_0 is irrational.

In order to make an $\mathrm{SL}(2,\mathbb{Z})$ transformation on this configuration we need to calculate $|\lambda|$ first. From the forms of e^ϕ and χ in (5.7), we get

$$|\lambda| = \left(r\chi_0 + s + \frac{r}{G_o^2 l} \right) \frac{\hat{h}''^{1/2}}{h''^{1/2}}\,, \tag{5.8}$$

where $\hat{h}'' = 1 + \hat{a}^2 u^2$. The parameter $\hat{a}^2$ is defined in terms of $\tilde{a}^2$ as follows,

$$\hat{a}^2 = \frac{(r\chi_0 + s)^2}{\left(r\chi_0 + s + \frac{r}{G_o^2 l} \right)^2} \tilde{a}^2\,. \tag{5.9}$$

Substituting $|\lambda|$ (given in (5.8)) and noting that $|\lambda_0| = (r\chi_0 + s)$ for NCOS limit we find the $\mathrm{SL}(2,\mathbb{Z})$ transformed metric from (5.4) as,

$$\begin{aligned} d\hat{s}^2 &= \epsilon \hat{l} h'^{1/2} \hat{h}''^{1/2} \times \\ &\quad\times \left[-(d\tilde{x}^0)^2 + (d\tilde{x}^1)^2 + h'^{-1} \sum_{i=2}^{5} (d\tilde{x}^i)^2 + \frac{(\hat{M} - \hat{\chi}_0 \hat{N})\hat{\alpha}'_{\rm eff}}{u^2}(du^2 + u^2 d\Omega_3^2) \right]. \end{aligned} \tag{5.10}$$

[7] Since the forms of the metric and dilaton essentially determine the nature of the decoupled theory, we will not give the explicit forms of other fields. The axion is required for the $\mathrm{SL}(2,\mathbb{Z})$ transformation.

Similarly the dilaton can be obtained from (5.2) as,

$$e^{\hat{\phi}} = \frac{\hat{G}_o^2 \hat{l}^2 \hat{h}''}{au}, \tag{5.11}$$

where we have defined the $\mathrm{SL}(2,\mathbb{Z})$ transformed parameters $\hat{G}_o^2$, $\hat{l}$, $\hat{\alpha}'_{\text{eff}}$ and $(\hat{M} - \hat{\chi}_0 \hat{N})$ as follows,

$$\begin{aligned} \hat{G}_o^2 &= (r\chi_0 + s)^2 G_o^2, \qquad \hat{l} = \frac{\left(r\chi_0 + s + \frac{r}{G_o^2 l}\right)}{(r\chi_0 + s)} l \\ \hat{\alpha}'_{\text{eff}} &= (r\chi_0 + s)\alpha'_{\text{eff}}, \qquad (\hat{M} - \hat{\chi}_0 \hat{N}) = \frac{(M - \chi_0 N)}{(r\chi_0 + s)}. \end{aligned} \tag{5.12}$$

Comparing (5.10) and (5.11) with the metric and dilaton in (5.7) we find that they have precisely the same form and thus we conclude that when χ_0 is irrational an $\mathrm{SL}(2,\mathbb{Z})$ transformation on NCOS theory gives another NCOS theory with different parameters related to the old parameters by eq. (5.12).

We would like to make a few comments here. First of all, note that the scaling parameter ϵ which is equal to $\cos\psi$ is $\mathrm{SL}(2,\mathbb{Z})$ invariant and so, the coordinates $\tilde{x}^{0,1,\dots,5}$ have the same form as the original NCOS gravity dual configuration in (5.7). Also, the parameter $\tilde{a}^2$ as well as h'' transforms under $\mathrm{SL}(2,\mathbb{Z})$ according to (5.9) while a^2 and h' are $\mathrm{SL}(2,\mathbb{Z})$ invariant. This can be understood from (2.2) since H and H' are $\mathrm{SL}(2,\mathbb{Z})$ invariant, but H'' is not. Also note that the parameter l which is proportional to $\cos\varphi$ is not $\mathrm{SL}(2,\mathbb{Z})$ invariant, but transforms according to eq. (5.12). Furthermore, we point out that the combination $(M - \chi_0 N)\alpha'_{\text{eff}}$ is $\mathrm{SL}(2,\mathbb{Z})$ invariant, but separately they transform according to (5.12). From the transformation of χ_0, it can be easily checked that if we start with an irrational χ_0, the transformed NCOS theory will also have irrational χ_0. The relations between the coupling constants and the length scales of the two NCOS theories are also given in (5.12). Lastly, we observe from eq. (5.8) that there are two special cases which may arise. Case (a) $(r\chi_0 + s) = 0$ i.e. χ_0 is rational. In this case, the transformed theory (5.10) and (5.11) reduces to OD1 theory as expected and studied in section 4. Case (b) $\left(r\chi_0 + s + \frac{r}{G_o^2 l}\right) = 0$. In this case, the parameter l of the transformed theory $(\hat{l})$ vanishes, which means $\cos\varphi$ vanishes. This is precisely the NCOS theory we obtained as the decoupling limit of (F, D5) configuration studied in subsection 3.1.

5.2 OD1 limit and $\mathrm{SL}(2,\mathbb{Z})$ duality

We have seen in the previous subsection that when χ_0 is irrational an NCOS theory will always go over to another NCOS theory with different parameters under $\mathrm{SL}(2,\mathbb{Z})$ transformation. This will not be true for OD1 theory as we will see in this subsection. Again we start with the supergravity configuration in (2.1). Under the OD1 decoupling limit eqs. (3.11) and (4.5) the harmonic functions take the forms,

$$H = 1 + \frac{N\alpha'}{\cos\varphi\cos\psi r^2} = \frac{1}{c^2\epsilon^2\tilde{u}^2},$$

$$
\begin{aligned}
H' &= 1 + \frac{\cos\psi N\alpha'}{\cos\varphi r^2} = \frac{f'}{c^2\tilde{u}^2}\,,\\
H'' &= 1 + \frac{\cos\varphi N\alpha'}{\cos\psi r^2} = \frac{1}{c^2\epsilon^2\tilde{u}^2}\,.
\end{aligned}
\tag{5.13}
$$

Note that in this case the harmonic functions have exactly the same forms as in the case of OD1 theory with rational χ_0 given in eq. (4.7). Of course, the explicit form of the angle $\cos\varphi$ in eq. (2.3) is different in this case. The parameters c^2 and f' are as defined before. The metric, dilaton and the axion in the decoupling limit have the forms,

$$
\begin{aligned}
d\bar{s}^2 &= \epsilon f'^{1/2} G^2_{o(1)}\left[-(d\bar{x}^0)^2 + (d\bar{x}^1)^2 + f'^{-1}\sum_{i=2}^{5}(d\bar{x}^i)^2 + \frac{N\tilde{\alpha}'_{\rm eff}}{\tilde{u}^2}(d\tilde{u}^2 + \tilde{u}^2 d\Omega_3^2)\right]\\
e^{\bar{\phi}} &= \frac{G^2_{o(1)}}{c\tilde{u}}, \qquad \bar{\chi} = \frac{\tilde{l}}{G^2_{o(1)}} + \chi_0\,.
\end{aligned}
\tag{5.14}
$$

The coordinates $\bar{x}^{0,1,\ldots,5}$ are the same as in (3.14). For SL$(2,\mathbb{Z})$ transformation we calculate $|\lambda|$ as before and it has the form

$$
|\lambda| = \left(r\chi_0 + s + \frac{r\tilde{l}}{G^2_{o(1)}}\right) f''^{1/2}\,,
\tag{5.15}
$$

where we have defined

$$
f'' = \left[1 + \frac{r^2}{\left(r\chi_0 + s + \frac{r\tilde{l}}{G^2_{o(1)}}\right)^2}\frac{c^2\tilde{u}^2}{G^4_{o(1)}}\right].
\tag{5.16}
$$

Now if we define new parameters l and $\tilde{c}$ by the following relations

$$
\begin{aligned}
r^2 l^2 &= \left(r\chi_0 + s + \frac{r\tilde{l}}{G^2_{o(1)}}\right)^2 G^4_{o(1)}\,,\\
\tilde{c}^2 &= \frac{c^2}{l^2}\,,
\end{aligned}
\tag{5.17}
$$

then f'' in eq. (5.16) becomes $f'' = 1+\tilde{c}^2\tilde{u}^2$ and the SL$(2,\mathbb{Z})$ transformd metric and dilaton become

$$
\begin{aligned}
d\hat{s}^2 &= \epsilon l f'^{1/2} f''^{1/2}\left[-(d\bar{x}^0)^2 + (d\bar{x}^1)^2 + f'^{-1}\sum_{i=2}^{5}(d\bar{x}^i)^2 + \frac{Nr\tilde{\alpha}'_{\rm eff}}{\tilde{u}^2}(d\tilde{u}^2 + \tilde{u}^2 d\Omega_3^2)\right],\\
e^{\hat{\phi}} &= f''\frac{r^2 l^2}{G^2_{o(1)} c\tilde{u}}\,,
\end{aligned}
\tag{5.18}
$$

where we have scaled the coordinates $\bar{x}^{0,1,\ldots,5} \to \sqrt{r}\bar{x}^{0,1,\ldots,5}$. Now comparing (5.18) with the metric and dilaton given in eq. (4.4) we notice that if the energy parameters $\tilde{u}$ and

u satisfy the same relation given in (3.18) then we get precisely the NCOS supergravity configuration in (4.4). The parameters of these two theories are related as,

$$G_o^2 = \frac{r^2}{G_{o(1)}^2}\,, \qquad \text{and} \qquad G_o^2 \alpha'_{\text{eff}} = r\tilde{\alpha}'_{\text{eff}}\,. \tag{5.19}$$

One thing to notice here is that, we started out with an OD1 theory with irrational χ_0, but after $\mathrm{SL}(2,\mathbb{Z})$ transformation we get an NCOS theory with rational[8] χ_0 (since (5.18) is the NCOS supergravity configuration for rational χ_0). Also, since NCOS theory with rational χ_0 is S-dual to OD1 theory with also rational χ_0 (discussed in section 4), so OD1 theory with irrational χ_0 and rational χ_0 are equivalent.

6. (F, D1, NS5, D5) solution, OD5 limit and $\mathrm{SL}(2,\mathbb{Z})$ duality

So far we have seen how NCOS and OD1 theory arise from the decoupling limit of (F, D1, NS5, D5) supergravity configuration and studied various properties of them. In this section we will discuss OD5 limit i.e. how OD5 theory also arises from a decoupling limit of the same supergravity configuration. We will first consider OD5 limit for rational χ_0 ($\chi_0 = 0$) and study S-duality and then consider the same limit for irrational χ_0 ($\chi_0 \neq 0$) and study the general $\mathrm{SL}(2,\mathbb{Z})$ transformation properties as in the previous sections.

6.1 OD5 limit for rational χ_0 and S-duality

Since χ_0 is rational we can set it to zero by an $\mathrm{SL}(2,\mathbb{Z})$ transformation. So, the supergravity solution we take is (2.1) with $\chi_0 = 0$. OD5 limit is taken by defining a dimensionless scaling parameter ϵ with $\cos\varphi = \epsilon \to 0$, keeping the following quantities fixed,

$$\tilde{\alpha}'_{\text{eff}} = \frac{\alpha'}{\epsilon}\,, \qquad G_{o(5)}^2 = \epsilon g_s\,, \qquad u = \frac{r}{\epsilon\tilde{\alpha}'_{\text{eff}}}\,, \qquad \cos\psi = l = \text{finite} < 1\,. \tag{6.1}$$

In the above $G_{o(5)}^2$ is the coupling constant of OD5 theory and $\tilde{\alpha}'_{\text{eff}}$ is the length scale. Note that the scaling parameter ϵ and the parameter l above have nothing to do with the parameters defined in the earlier sections. Also the case $l = 1$ corresponds to $k = 0$ (see eq. (2.3)) and so (F, D1) strings are absent and this case has already been studied before in [4, 9, 19]. In the decoupling limit (6.1) the harmonic functions take the forms,

$$\begin{aligned} H &= 1 + \frac{N\alpha'}{\cos\varphi\cos\psi r^2} = \frac{1}{d^2\epsilon^2u^2}\,,\\ H' &= 1 + \frac{\cos\psi N\alpha'}{\cos\varphi r^2} = \frac{1}{\tilde{d}^2\epsilon^2u^2}\,,\\ H'' &= 1 + \frac{\cos\varphi N\alpha'}{\cos\psi r^2} = \frac{F''}{d^2u^2}\,, \end{aligned} \tag{6.2}$$

where $d^2 = \tilde{\alpha}'_{\text{eff}}l/N$, $\tilde{d}^2 = \tilde{\alpha}'_{\text{eff}}/(Nl)$ and $F'' = 1 + d^2u^2$, with N representing the number of NS5 branes. Therefore, $d\tilde{d} = \tilde{\alpha}'_{\text{eff}}/N$ and $d/\tilde{d} = l$. The gravity dual configuration of OD5

[8] This can also be understood from the $\mathrm{SL}(2,\mathbb{Z})$ transformation of χ_0 and g_s given in (5.2) for OD1 limit.

theory obtained from (2.1) is

$$
\begin{aligned}
ds^2 &= \epsilon F''^{1/2}\left[-(d\tilde{x}^0)^2+\sum_{i=1}^{5}(d\tilde{x}^i)^2+\frac{N\tilde{\alpha}'_{\text{eff}}}{u^2}\left(du^2+u^2d\Omega_3^2\right)\right],\\
e^{\phi} &= \frac{G_{o(5)}^2F''}{du}\,, \qquad \chi=\frac{1}{G_{o(5)}^2F''}\,,\\
B_{01} &= \frac{\sqrt{1-l^2}}{l}\epsilon^2d^2u^2\,, \qquad F_{NS}=2N\alpha'\epsilon_3\,,\\
C_{01} &= \frac{\epsilon^4\sqrt{1-l^2}}{lG_{o(5)}^2}d^2u^2\,, \qquad F_{RR}=-2M\alpha'\epsilon_3\,,\\
C_{2345} &= \frac{\epsilon l\sqrt{1-l^2}}{G_{o(5)}^2}\,,
\end{aligned}
\tag{6.3}
$$

where the fixed coordinates are defined as

$$
\tilde{x}^{0,1}=\sqrt{l}x^{0,1}\,, \qquad \tilde{x}^{2,\ldots,5}=\frac{1}{\sqrt{l}}x^{2,\ldots,5}\,, \tag{6.4}
$$

where M is the number of D5 branes. This is the gravity dual description of OD5 theory. The S-dual configuration of OD5 theory can be obtained using the general relations given in (4.9). We calculate $|\lambda|=[\chi^2+e^{-2\phi}]^{1/2}$ from (6.3) for this purpose as,

$$
|\lambda|=\frac{1}{F''^{1/2}G_{o(5)}^2}\,. \tag{6.5}
$$

Also insisting that the transformed metric be asymptotically minkowskian such that $d\hat{s}^2=(|\lambda|/|\lambda_0|)ds^2$, with $|\lambda_0|=\epsilon/G_{o(5)}^2$, we get the S-dual configuration as,

$$
\begin{aligned}
d\hat{s}^2 &= -(d\tilde{x}^0)^2+\sum_{i=1}^{5}(d\tilde{x}^i)^2+\frac{N\tilde{\alpha}'_{\text{eff}}}{u^2}\left(du^2+u^2d\Omega_3^2\right),\\
e^{\hat{\phi}} &= \frac{1}{G_{o(5)}^2du}\,, \qquad \hat{\chi}=-G_{o(5)}^2=-\frac{N}{M}=\text{a rational no.}\,,\\
\hat{B}_{01} &= -C_{01}\,, \qquad \hat{F}_{NS}=-F_{RR}\,,\\
\hat{C}_{01} &= B_{01}\,, \qquad \hat{F}_{RR}=F_{NS}\,,\\
\hat{C}_{2345} &= C_{2345}\,.
\end{aligned}
\tag{6.6}
$$

We note from above that in the UV, $e^{\hat{\phi}}\ll 1$ and for $N\gg 1$, the curvature remains small and therefore we have a valid supergravity description. By comparison [28, 29], we find that this is precisely the supergravity dual of LST, where the closed string coupling $g_s=\epsilon/G_{o(5)}^2\to 0$ and $\tilde{\alpha}'_{\text{ef}}=$ finite (the length scale of LST). Thus we conclude that OD5 theory goes over to LST under type IIB S-duality.

6.2 OD5 limit for irrational χ_0 and $\mathrm{SL}(2,\mathbb{Z})$ duality

In this case $\chi_0\neq 0$ in the (F, D1, NS5, D5) bound state given in (2.1). Under OD5 limit (6.1), the harmonic functions take exactly the same form as given in (6.2), although the explicit form of $\cos\varphi$ is different for this case. So, the metric and dilaton have exactly the same form as in (6.3), but the axion is modified to include χ_0 term[9]. So,

$$\chi = \frac{1}{G^2_{o(5)} F''} + \chi_0 \,. \tag{6.7}$$

The $\mathrm{SL}(2,\mathbb{Z})$ transformation of various fields are given in (5.1)–(5.4). For this purpose we calculate,

$$|\lambda| = |r\tau + s| = \left(r\chi_0 + s + \frac{r}{G^2_{o(5)}}\right) \frac{\hat{F}''^{1/2}}{F''^{1/2}}\,, \tag{6.8}$$

where $\hat{F}'' = 1 + \hat{d}^2 u^2$. The parameter $\hat{d}^2$ is given in terms of d^2 as,

$$\hat{d}^2 = \frac{(r\chi_0 + s)^2}{\left(r\chi_0 + s + \frac{r}{G^2_{o(5)}}\right)^2} d^2 \,. \tag{6.9}$$

The $\mathrm{SL}(2,\mathbb{Z})$ transformed metric and dilaton can be obtained from (5.4) and (5.2) as,

$$\begin{aligned} d\hat{s}^2 &= \hat{\epsilon}\hat{F}''^{1/2}\left[-(d\tilde{x}^0)^2 + \sum_{i=1}^{5}(d\tilde{x}^i)^2 + \frac{\hat{N}\hat{\tilde{\alpha}}'_{\mathrm{eff}}}{u^2}\left(du^2 + u^2 d\Omega_3^2\right)\right], \\ e^{\hat{\phi}} &= \frac{\hat{G}^2_{o(5)}\hat{F}''}{\hat{d}u}\,, \end{aligned} \tag{6.10}$$

where the $\mathrm{SL}(2,\mathbb{Z})$ transformed parameters are given as,

$$\begin{aligned} \hat{\epsilon} &= \frac{\left(r\chi_0 + s + \frac{r}{G^2_{o(5)}}\right)}{(r\chi_0 + s)}\epsilon\,, \qquad \hat{\tilde{\alpha}}'_{\mathrm{eff}} = \frac{(r\chi_0 + s)}{\left(r\chi_0 + s + \frac{r}{G^2_{o(5)}}\right)}\tilde{\alpha}'_{\mathrm{eff}}\,, \\ \hat{N} &= \frac{\left(r\chi_0 + s + \frac{r}{G^2_{o(5)}}\right)}{(r\chi_0 + s)}N\,, \qquad \hat{G}^2_{o(5)} = \left(r\chi_0 + s + \frac{r}{G^2_{o(5)}}\right)(r\chi_0 + s)G^2_{o(5)}\,. \end{aligned} \tag{6.11}$$

Comparing (6.10) with the metric and dilaton in (6.3), we find that they have exactly the same form and therefore we conclude that for χ_0 = irrational an OD5 theory goes over to another OD5 theory under $\mathrm{SL}(2,\mathbb{Z})$ transformation. Notice that since $l = \cos\psi$ is $\mathrm{SL}(2,\mathbb{Z})$ invariant the coordinates $\tilde{x}^{0,1,\ldots,5}$ remain the same in the two OD5 theories. However, the scaling parameter $\epsilon = \cos\varphi$ transforms according to (6.11). Also, since H and H' are $\mathrm{SL}(2,\mathbb{Z})$ invariant, so both d and $\tilde{d}$ must transform in the same way as given in (6.9) and ϵ must transform in the opposite way to d and $\tilde{d}$ as obtained in (6.11). The combination $N\tilde{\alpha}'_{\mathrm{eff}}$ is $\mathrm{SL}(2,\mathbb{Z})$ invariant. The coupling constant $G^2_{o(5)}$ and the length scale $\tilde{\alpha}'_{\mathrm{eff}}$ of the $\mathrm{SL}(2,\mathbb{Z})$ transformed OD5 theory are given in (6.11). From eq. (6.8) we find that when $r\chi_0 + s = 0$ i.e. when χ_0 is rational, the $\mathrm{SL}(2,\mathbb{Z})$ transformed configuration (6.10) reduces precisely to LST studied in the previous subsection as expected. However, $r\chi_0 + s + r/G^2_{o(5)}$ can not be zero in this case because that would imply $\hat{\epsilon}$ to be exactly equal to zero and there would be no scaling parameter to obtain a decoupled theory.

[9]Here also we do not give the explicit forms of other gauge fields since we will only indicate the nature of the $\mathrm{SL}(2,\mathbb{Z})$ transformed theory.

7. Conclusion

To summarize, starting from an $SL(2,\mathbb{Z})$ invariant bound state (F, D1, NS5, D5) of type IIB supergravity we have shown how to take various decoupling limits leading to the supergravity duals of NCOS theory, OD1 theory and OD5 theory. The decoupled theories obtained in this way are in general different from those obtained by taking low energy limits on (F, D5), (D1, NS5) and (NS5, D5) separately in the sense that their field contents are different. As a result, the decoupled theories or their supergravity duals can be subjected to the full $SL(2,\mathbb{Z})$ duality of type IIB supergravity. We have studied the $SL(2,\mathbb{Z})$ transformation properties of various (5+1) dimensional decoupled theories obtained from this bound state. The asymptotic value of the RR scalar or axion present in the supergravity configuration is crucial to determine the behavior of the decoupled theories under the $SL(2,\mathbb{Z})$ transformation. We found that when χ_0 is rational the general NCOS and OD1 theories are related by the strong-weak duality as obtained before in the special cases i.e. for (F, D5) and (D1, NS5) solutions in [4, 11]. But when χ_0 is irrational NCOS theory gives another NCOS theory with different values of the parameters. However, for OD1 theory we surprisingly found a different conclusion that even for irrational χ_0, OD1 theory goes over to an NCOS theory under $SL(2,\mathbb{Z})$ transformation. This shows that OD1 theories with rational and irrational χ_0 are equivalent. We have also addressed the question of self-duality of both (5+1) dimensional NCOS and OD1 theories in this context. We have been able to show the self-duality only in the region where the coupling constant blows up and the supergravity description breaks down. We mentioned that this result can be interpreted in two ways. It can either mean that the self-duality conjecture can not be tested at the level of supergravity dual or it can be taken as a supporting evidence for the self-duality conjecture if the supergravity description somehow remains valid in the strongly coupled region due to some underlying non-renormalization effect. For completeness, we have also studied the $SL(2,\mathbb{Z})$ transformation on OD5 theories. Here we found that for rational χ_0, OD5 is related to LST by the S-duality of type IIB theory, but for irrational χ_0, OD5 theory gives another OD5 theory with different values of the parameters. In conclusion, we point out that since type IIB string theory is believed to possess an $SL(2,\mathbb{Z})$ symmetry, by working with the $SL(2,\mathbb{Z})$ invariant supergravity configuration, we have been able to access the full $SL(2,\mathbb{Z})$ group of transformation to relate various decoupled theories in (5+1) dimensions.

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References

[1] O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

[2] U.H. Danielsson, A. Guijosa and M. Kruczenski, *IIA/B, wound and wrapped*, *J. High Energy Phys.* **0** (2000) 020 [hep-th/0009182].

[3] J. Maldacena, *The large N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[4] R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, *OM theory in diverse dimensions*, *J. High Energy Phys.* **8** (2000) 008 [hep-th/0006062].

[5] J.X. Lu and S. Roy, *Non-threshold (F,Dp) bound states*, *Nucl. Phys.* **B 560** (1999) 181 [hep-th/9904129].

[6] T. Harmark, *Supergravity and space-time non-commutative open string theory*, *J. High Energy Phys.* **7** (2000) 043 [hep-th/0006023].

[7] N. Seiberg, L. Susskind and N. Toumbas, *Strings in background electric field, space/time noncommutativity and a new noncritical string theory*, *J. High Energy Phys.* **6** (2000) 021 [hep-th/0005040].

[8] R. Gopakumar, J. Maldacena, S. Minwalla and A. Strominger, *S-duality and noncommutative gauge theory*, *J. High Energy Phys.* **6** (2000) 036 [hep-th/0005048].

[9] I. Mitra and S. Roy, *(NS5, Dp) and (NS5, D$(p+2)$, Dp) bound states of type IIB and type IIA string theories*, *J. High Energy Phys.* **2** (2001) 026 [hep-th/0011236].

[10] M. Cederwall, U. Gran, M. Nielsen and B.E.W. Nilsson, (p,q) *5-branes in non-zero b-field*, *J. High Energy Phys.* **1** (2000) 037 [hep-th/9912106].

[11] T. Harmark, *Open branes in space-time non-commutative little string theory*, *Nucl. Phys.* **B 593** (2001) 76 [hep-th/0007147].

[12] M. Alishahiha, Y. Oz and J.G. Russo, *Supergravity and light-like non-commutativity*, *J. High Energy Phys.* **9** (2000) 002 [hep-th/0007215].

[13] M. Alishahiha, *On type II NS5-branes in the presence of an RR field*, hep-th/0002198.

[14] M. Berkooz, M. Rozali and N. Seiberg, *On transverse fivebranes in M(atrix) theory on* t^5, *Phys. Lett.* **B 408** (1997) 105 [hep-th/9704089].

[15] N. Seiberg, *New theories in six dimensions and matrix description of M-theory on* T^5 *and* $T^5/\mathbb{Z}_2$, *Phys. Lett.* **B 408** (1997) 98 [hep-th/9705221].

[16] R. Dijkgraaf, E. Verlinde and H. Verlinde, *BPS quantization of the five-brane*, *Nucl. Phys.* **B 486** (1997) 89 [hep-th/9604055].

[17] R. Dijkgraaf, E. Verlinde and H. Verlinde, *5d black holes and matrix strings*, *Nucl. Phys.* **B 506** (1997) 121 [hep-th/9704018].

[18] A. Losev, G.W. Moore and S.L. Shatashvili, *M & m's*, *Nucl. Phys.* **B 522** (1998) 105 [hep-th/9707250].

[19] M. Alishahiha and Y. Oz, *Supergravity and "new" six-dimensional gauge theories*, *Phys. Lett.* **B 495** (2000) 418 [hep-th/0008172].

[20] J.X. Lu and S. Roy, *(F,D5) bound state,* $\mathrm{SL}(2,\mathbb{Z})$ *invariance and the descendant states in type IIB/A string theory*, *Phys. Rev.* **D 60** (1999) 126002 [hep-th/9905056].

[21] J.X. Lu, S. Roy and H. Singh, *Yang-Mills theory*, *J. High Energy Phys.* **9** (2000) 020 [hep-th/0006193].

[22] J.X. Lu, S. Roy and H. Singh, $\mathrm{SL}(2,\mathbb{Z})$ *duality and 4-dimensional noncommutative theories*, *Nucl. Phys.* **B 595** (2001) 298 [hep-th/0007168].

[23] J.G. Russo and M.M. Sheikh-Jabbari, *On noncommutative open string theories*, *J. High Energy Phys.* **7** (2000) 052 [hep-th/0006202].

[24] R.G. Cai and N. Ohta, *(F1, D1, D3) bound state, its scaling limits and* $\mathrm{SL}(2,\mathbb{Z})$ *duality*, *Prog. Theor. Phys.* **104** (2000) 1073 [hep-th/0007106].

[25] I. Mitra and S. Roy, *(NS5, D5, D3) bound state, OD3, OD5 limits and* $\mathrm{SL}(2,\mathbb{Z})$ *duality*, hep-th/0107127.

[26] J.X. Lu, *(1+p)-dimensional open D(p− 2) brane theories*, *J. High Energy Phys.* **8** (2001) 049 [hep-th/0102056].

[27] H. Larsson and P. Sundell, *Open string/open D-brane dualities: old and new*, *J. High Energy Phys.* **6** (2001) 008 [hep-th/0103188].

[28] O. Aharony, M. Berkooz, D. Kutasov and N. Seiberg, *Linear dilatons, NS5-branes and holography*, *J. High Energy Phys.* **0** (1998) 004 [hep-th/9808149].

[29] N. Itzhaki, J.M. Maldacena, J. Sonnenschein and S. Yankielowicz, *Supergravity and the large-N limit of theories with sixteen supercharges*, *Phys. Rev.* **D 58** (1998) 046004 [hep-th/9802042].